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Institution: Beijing-Dublin International College
Module: University Physics 2 (BDIC2008J)
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Problem Set 8

Optical Wave Interference

Problem 1. *The wavelength of yellow sodium light in air is* 589 nm*.* ***(a)*** *What is its frequency?* ***(b)*** *What is its wavelength in glass whose index of refraction is 1.52?* ***(c)*** *From the results of (a) and (b), find its speed in this glass.*

Solution. **(a)** The frequency of yellow sodium light is

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^{8}\,\mathrm{m/s}}{589 \times 10^{-9}\,\mathrm{m}} = 5.09 \times 10^{14}\,\mathrm{Hz}$$

(b) When traveling through the glass, its wavelength is

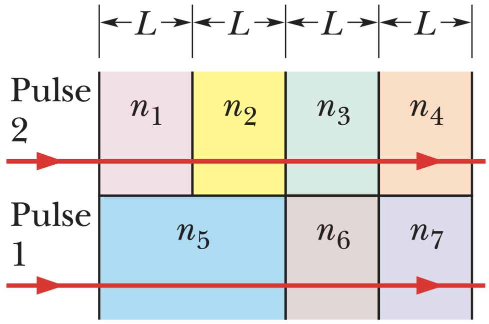
$$\lambda_n = \frac{\lambda}{n} = \frac{589\,\mathrm{nm}}{1.52} = 388\,\mathrm{nm}$$

(c) The light speed when traveling through the glass is

$$v = f\lambda_n = (5.09 \times 10^{14}\,\mathrm{Hz})(388 \times 10^{-9}\,\mathrm{m}) = 1.97 \times 10^{8}\,\mathrm{m/s}$$

□

Problem 2. *In the figure below, two light pulses are sent through layers of plastic with thicknesses of either L or $2L$ as shown and indexes of refraction $n_1 = 1.55$, $n_2 = 1.70$, $n_3 = 1.60$, $n_4 = 1.45$, $n_5 = 1.59$, $n_6 = 1.65$, and $n_7 = 1.50$.* ***(a)*** *Which pulse travels through the plastic in less time?* ***(b)*** *What is the difference in the traversal times of the pulses in multiple L/c?*



Solution. **(a)** The time t_2 it takes for pulse 2 to travel through the plastic is

$$t_2 = \frac{L}{c/1.55} + \frac{L}{c/1.70} + \frac{L}{c/1.60} + \frac{L}{c/1.45} = \frac{6.30L}{c}$$

Similarly for pulse 1:

$$t_1 = \frac{2L}{c/1.59} + \frac{L}{c/1.65} + \frac{L}{c/1.50} = \frac{6.33L}{c}$$

Thus, pulse 2 travels through the plastic in less time.

(b) The time difference (as a multiple of L/c) is

$$\Delta = t_2 - t_1 = \frac{6.33L}{c} - \frac{6.30L}{c} = \frac{0.03L}{c}$$

Thus, the multiple is 0.03. □

Problem 3. *In a double-slit arrangement the slits are separated by a distance equal to 100 times the wavelength of the light passing through the slits.* ***(a)*** *What is the angular separation in radians between the central maximum and an adjacent maximum?* ***(b)*** *What is the distance between these maxima on a screen* $50.0\,\mathrm{cm}$ *from the slits?*

Solution. **(a)** For the maximum adjacent to the central one, we set $m = 1$ in optical path difference $\Delta L = d \sin\theta = \pm m\lambda$ and obtain

$$\theta_1 = \arcsin\left(\frac{m\lambda}{d}\right)\Bigg|_{m=1} = \arcsin\left[\frac{(1)(\lambda)}{100\lambda}\right] = 0.010\,\mathrm{rad}$$

(b) Since $x_1 = D\tan\theta_1$, we obtain

$$x_1 = (500\,\mathrm{mm})\tan(0.010\,\mathrm{rad}) = 5.0\,\mathrm{mm}$$

The separation is $\Delta x = x_1 - x_0 = x_1 - 0 = 5.0\,\mathrm{mm}$. $\square$

Problem 4. *A double-slit arrangement produces interference fringes for sodium light* ($\lambda = 589\,\mathrm{nm}$) *that have an angular separation of* $3.50 \times 10^{-3}\,\mathrm{rad}$. *For what wavelength would the angular separation be 10.0% greater?*

Solution. The interference at a point depends on the optical path difference of the light rays reaching that point from the two slits.

The angular positions of the maxima of a two-slit interference pattern are $\Delta L = d \sin\theta = m\lambda$, where ΔL is the optical path difference, d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$ to good approximation. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$.

Let λ' be the wavelength for which the angular separation is greater by 10.0%. Then, $1.10\lambda/d = \lambda'/d$. or

$$\lambda' = 1.10\lambda = 1.10(589\,\mathrm{nm}) = 648\,\mathrm{nm}$$

The angular separation $\Delta\theta$ is proportional to the wavelength of the light. For small θ, we have

$$\Delta\theta' = \left(\frac{\lambda'}{\lambda}\right)\Delta\theta$$

□

Problem 5. *A double-slit arrangement produces interference fringes for sodium light* ($\lambda = 589\,\mathrm{nm}$) *that are* $0.20°$ *apart. What is the angular separation if the arrangement is immersed in water* ($n = 1.33$)*?*

Solution. The distance between adjacent maxima is given by $\Delta x = \lambda D/d$. Dividing both sides by D, this becomes $\Delta\theta = \lambda/d$ with θ in radians. In the steps that follow, however, we will end up with an expression where degrees may be directly used. Thus, in the present case,

$$\Delta\theta_n = \frac{\lambda_n}{d} = \frac{\lambda}{nd} = \frac{\Delta\theta}{n} = \frac{0.20°}{1.33} = 0.15°$$

□

Problem 6. *Suppose that Young's experiment is performed with blue-green light of wavelength* 500 nm*. The slits are* 1.20 mm *apart, and the viewing screen is* 5.40 m *from the slits. How far apart are the bright fringes near the centre of the interference pattern?*

Solution. The condition for a maximum in the two-slit interference pattern is $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction.

If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$, and the angular separation of adjacent maxima, one associated with the integer m and the other associated with the integer $m+1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance D away is given by

$$\Delta x = D\Delta\theta = \lambda D/d$$

Thus,

$$\Delta x = \frac{(500 \times 10^{-9}\,\mathrm{m})(5.40\,\mathrm{m})}{1.20 \times 10^{-3}\,\mathrm{m}} = 2.25 \times 10^{-3}\,\mathrm{m} = 2.25\,\mathrm{mm}$$

For small θ, the spacing is nearly uniform. However, away from the centre of the pattern, θ increases and the spacing gets larger. □

Problem 7. *In a double-slit experiment, the distance between slits is* 5.0 mm *and the slits are* 1.0 m *from the screen. Two interference patterns can be seen on the screen: one due to light of wavelength* 480 nm*, and the other due to light of wavelength* 600 nm*. What is the separation on the screen between the third-order* ($m = 3$) *bright fringes of the two interference patterns?*

Solution. The maxima of a two-slit interference pattern are at angles θ given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be replaced by θ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of m is

$$\Delta\theta = (m/d)(\lambda_2 - \lambda_1)$$

and their separation on a screen a distance D away is

$$\begin{aligned}\Delta x = D \tan\Delta\theta \approx D\Delta\theta &= \left(\frac{mD}{d}\right)(\lambda_2 - \lambda_1) \\ &= \left[\frac{3(1.0\,\mathrm{m})}{5.0 \times 10^{-3}\,\mathrm{m}}\right]\left(600 \times 10^{-9}\,\mathrm{m} - 480 \times 10^{-9}\,\mathrm{m}\right) = 7.2 \times 10^{-5}\,\mathrm{m}\end{aligned}$$

The small angle approximation $\tan\theta \approx \Delta\theta$ (in radians) is made. $\square$

Problem 8. *We wish to coat flat glass* ($n = 1.50$) *with a transparent material* ($n = 1.25$) *so that reflection of light at wavelength* 600 nm *is eliminated by interference. What minimum thickness can the coating have to do this?*

Solution. For complete destructive interference, we want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of π rad.

Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of π rad on reflection. If L is the thickness of the coating, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_c)$, where λ_c is the wavelength in the coating. If n is the index of refraction of the coating, $\lambda_c = \lambda/n$, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi$$

for L. Here m is an integer. The result is $L = \dfrac{(2m+1)\lambda}{4n}$.

To find the least thickness for which destructive interference occurs, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{600 \times 10^{-9}\,\mathrm{m}}{4(1.25)} = 1.20 \times 10^{-7}\,\mathrm{m}$$

A light ray reflected by a material changes phase by π rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is travelling. □

Problem 9. *White light is sent downward onto a horizontal thin film that is sandwiched between two materials. The indexes of refraction are 1.80 for the top material, 1.70 for the thin film, and 1.50 for the bottom material. The film thickness is* 5.00×10^{-7} m. *Of the visible wavelengths* (400 *to* 700 nm) *that result in fully constructive interference at an observer above the film, which is the* ***(a)*** *longer and* ***(b)*** *shorter wavelength? The materials and film are then heated so that the film thickness increases.* ***(c)*** *Does the light resulting in fully constructive interference shift toward longer or shorter wavelengths?*

Solution. **(a)** We are dealing with a thin film (material 2) in a situation where $n_1 > n_2 > n_3$, looking for strong *reflections*. Therefore, with lengths in nm and $L = 500$ and $n_2 = 1.7$, we have

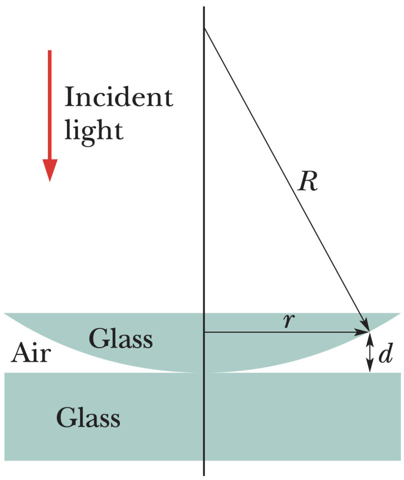
$$\lambda = \frac{2n_2L}{m} = \begin{cases} 1700 & \text{for m} = 1 \\ 850 & \text{for m} = 2 \\ 567 & \text{for m} = 3 \\ 425 & \text{for m} = 4 \end{cases}$$

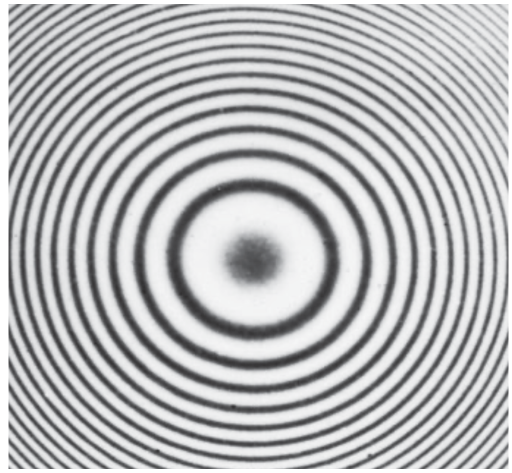
from which we see the latter two values are in the given range. The longer wavelength ($m = 3$) is $\lambda = 567\,\text{nm}$.

(b) The shorter wavelength ($m = 4$) is $\lambda = 425\,\text{nm}$.

(c) We assume the temperature dependence of the refraction index is negligible. From the proportionality evident in the part (a) equation, longer L means longer λ. □

Problem 10. *The left figure shows a lens with radius of curvature R lying on a flat glass plate and illuminated from above by light with wavelength λ. The right figure (a photograph taken from above the lens) shows that circular interference fringes (known as Newton's rings) appear, associated with the variable thickness d of the air film between the lens and the plate. Find the radii r of the interference maxima and minima assuming $r/R \ll 1$.*





Solution. The formation of Newton's rings is due to the interference between the rays reflected from the flat glass plate and the curved lens surface. Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is d, the condition for a maximum in intensity is $2d = (m + 1/2)\lambda$, where λ is the wavelength in air and m is an integer. Therefore,

$$d = (2m + 1)\lambda/4$$

As the geometry of the figure above, $d = R - \sqrt{R^2 - r^2}$, where R is the radius of curvature of the lens and r is the radius of a Newton's ring. Thus, $(2m+1)\lambda/4 = R - \sqrt{R^2 - r^2}$. First, we rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4}$$

Next, we square both sides, rearrange to solve for r^2, then take the square root. We get

$$r = \sqrt{\frac{(2m+1)R\lambda}{2} - \frac{(2m+1)^2\lambda^2}{16}}$$

If R is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{(2m+1)R\lambda}{2}}$$

Similarly, the radii of the dark fringes are given by

$$r = \sqrt{\frac{(2m)R\lambda}{2}} = \sqrt{mR\lambda}$$

□